

Influence of Fermion Velocity Renormalization on Dynamical Mass Generation in QED₃

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We study dynamical fermion mass generation in (2+1)-dimensional quantum electrodynamics with a gauge field coupling to massless Dirac fermions and non-relativistic scalar bosons. We calculate the fermion velocity renormalization and then examine its influence on dynamical mass generation by using the Dyson-Schwinger equation. It is found that dynamical mass generation takes place even after including the scalar bosons as long as the bosonic compressibility parameter ξ is sufficiently small. In addition, the fermion velocity renormalization enhances the dynamically generated mass.

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There are two basic reasons why quantum electrodynamics in (2+1) dimensions (QED₃) has been investigated extensively for nearly thirty years. Firstly, as a relatively simple gauge theory, QED₃ of massless fermion itself as a quantum field theory exhibits many interesting features, such as dynamical chiral symmetry breaking (DCSB) [1–20], asymptotic freedom [8], and fermion confinement [21–23], so the investigations of this model can shed some light on our understandings of quantum chromodynamics. Secondly, QED₃ and its non-relativistic variants have wide applications in several planar condensed-matter systems, including high- T_c superconductors [12, 17, 24–36] and fractional quantum Hall systems [37, 38].

Among the interesting features of QED₃, DCSB plays an important role and has been an active research field for more than two decades. One fascinating feature of DCSB is that it can generate fermion mass via fermion-antifermion condensation mediated by a strong gauge field without introducing Higgs particle. In their breakthrough work, Appelquist *et al* [3] predicted that DCSB occurs only when the fermion flavor is less than some critical value N_c by solving the Dyson-Schwinger (DS) equation. When $N > N_c$, the fermions remain massless and the chiral symmetry is preserved. Most analytical and numerical computations seem to agree that $N_c \approx 3.5$ [3–5, 14] despite some early controversies [6, 7].

DCSB in QED₃ provides an elegant field-theoretic description for some important phenomena of high- T_c cuprate superconductors. In undoped high- T_c superconductors, DCSB takes place since the physical flavor is $N = 2$. It is widely interpreted as the formation of long-range anti-ferromagnetic (AFM) order [13, 29, 31]. At finite doping, the dynamics of doped holes can be described by introducing additional scalar bosons within the slave-boson treatment of t - J model [27, 33]. At low temperature, these scalar bosons undergo Bose-Einstein condensation and consequently lead to superconductivity. An interesting question is how DCSB is affected by the additional scalar bosons. This question is not easy to answer because the scalar boson sector is not well understood [28–30]. Kim *et al* [29] argued that the only effect of non-relativistic scalar bosons is to statically screen the temporal component of gauge field. Based on this argument, they simply neglected both the scalar bosons and the temporal component of gauge field, and found that $N'_c = N_c/2 = 16/\pi^2$, implying that DCSB does not occur in the presence of scalar bosons [29]. Liu *et al* [12, 13, 17] also studied this problem, but found that DCSB can occur even if the gauge field couples to scalar bosons as long as the gauge field does not acquire a large mass via Anderson-Higgs mechanism. For calculational simplicity, they assumed that the scalar bosons are relativistic. However, in the realistic effective QED₃ theory of high- T_c superconductors, the scalar bosons should be non-relativistic [28, 29]. The breaking of Lorentz invariance due to non-relativistic scalar bosons can result in novel features, such as singular renormalization of fermion velocity [28] and non-Fermi liquid behaviors [28, 39], compared with the case of relativistic scalar bosons. Such features are not considered in the previous works [12, 13, 17, 29].

In this Letter, we would like to revisit this problem. Different from Kim *et al* and Liu *et al*, we will include explicitly the influence of non-relativistic scalar bosons on DCSB in QED₃ theory. Recently, it was found that the velocity renormalization can weaken or even destroy DCSB in the context of graphene [40, 41], where the fermion mass is generated by long-range Coulomb interaction. This motivates us to study the effects of fermion velocity renormalization on DCSB in the present QED₃ model. We first build a DS mass equation in the presence of both temporal and spatial components of gauge field, and then solve this equation after incorporating the fermion velocity renormalization. We found that the fermion velocity renormalization does not destroy DCSB in QED₃ and actually enhances the dynamical mass, which is quite different from that in graphene [40, 41].

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In the (2+1)-dimensional Euclidean space, the continuum effective Lagrangian is given by [28]

$$\mathcal{L} = \sum_{\sigma=1}^N \bar{\psi}_{\sigma} v_{\mu} \gamma^{\mu} (\partial_{\mu} + i a_{\mu}) \psi_{\sigma} + \phi^{*} (\partial_0 - \mu_B + i a_0) \phi - \frac{1}{2m_B} \phi^{*} (\nabla + i \mathbf{a})^2 \phi. \quad (1)$$

The massless Dirac fermions are described by a 4×1 spinor field ψ_{σ} , whose conjugate spinor field is defined as $\bar{\psi}_{\sigma} = \psi_{\sigma}^{\dagger} \gamma^0$ [28, 29, 33]. The 4×4 γ^{μ} obey the Clifford algebra $\{\gamma^{\mu}, \gamma^{\nu}\} = 2\delta^{\mu\nu}$ with $\mu, \nu = 0, 1, 2$, and for simplicity we take $v_{\mu} = 1$ for $\mu = 0$ and $v_{\mu} = v$ for $\mu = 1, 2$. In the context of high-temperature superconductors, the physical fermion flavor is actually $N = 2$, reflecting the two spin components [28, 29, 33]. At present, we consider a large flavor N in order to perform the $1/N$ expansion. The non-relativistic scalar boson field ϕ represents charge degree of freedom [28, 29, 33]. Both the Dirac fermion ψ and scalar boson ϕ interact with the gauge field a_{μ} whose temporal component is denoted as a_0 while spatial components \mathbf{a} , but there is no direct coupling between ψ and ϕ [28, 29, 33].

In general, the polarization tensor $\Pi_{\mu\nu}(q)$ can be conveniently decomposed in terms of two independent transverse tensors [26]

$$\Pi_{\mu\nu}(q) = \Pi_A(q) A_{\mu\nu}(q) + \Pi_B(q) B_{\mu\nu}(q), \quad (2)$$

where $A_{\mu\nu}(q) = \left(\delta_{\mu 0} - \frac{q_{\mu} q_0}{q^2}\right) \frac{q^2}{\mathbf{q}^2} \left(\delta_{\nu 0} - \frac{q_{\nu} q_0}{q^2}\right)$ and $B_{\mu\nu}(q) = \delta_{\mu i} \left(\delta_{ij} - \frac{q_i q_j}{\mathbf{q}^2}\right) \delta_{j\nu}$ with $A_{\mu\nu}(q)$ and $B_{\mu\nu}(q)$ orthogonal and satisfying $A_{\mu\nu}(q) + B_{\mu\nu}(q) = \delta_{\mu\nu} - \frac{q_{\mu} q_{\nu}}{q^2}$. Using these relations, the gauge field propagator can be recast in the form

$$D_{\mu\nu}(q) = \frac{A_{\mu\nu}(q)}{q^2 + \Pi_A(q)} + \frac{B_{\mu\nu}(q)}{q^2 + \Pi_B(q)}, \quad (3)$$

with $\Pi_A(q) = \frac{q^2}{\mathbf{q}^2} \Pi_{00}$ and $\Pi_B(q) = \Pi_{ii} - \frac{q_0^2}{\mathbf{q}^2} \Pi_{00}$.

In the effective QED₃ theory, there is no kinetic term $F_{\mu\nu} F^{\mu\nu}$, so the gauge field a_{μ} obtains its dynamics only after integrating out fermion and boson fields. In general, the gauge field propagator takes the form

$$D_{\mu\nu}(q) = \frac{A_{\mu\nu}(q)}{\Pi_A^f(q) + \Pi_A^b(q)} + \frac{B_{\mu\nu}(q)}{\Pi_B^f(q) + \Pi_B^b(q)}. \quad (4)$$

Here, $\Pi^f(q)$ and $\Pi^b(q)$ are the polarization functions contributed by the massless Dirac fermion and scalar boson. In a strict sense, one need to calculate these two polarization functions explicitly. It is technically quite easy to compute $\Pi^f(q)$, whereas hard to compute $\Pi^b(q)$. As demonstrated in Ref. [28, 29], the finite compressibility of scalar bosons ensures that, $\Pi_A^b(q = 0) \neq 0$, therefore the temporal component of gauge field is statically screened and becomes massive. This process can be described by the following approximation

$$\Pi_A^b(q) \approx \xi, \quad (5)$$

with ξ being a phenomenological parameter. In the absence of a detailed understanding of the boson sector, we follow the assumption of Ref. [28, 29] that the transverse gauge propagator is dominated by the fermion part,

$$\Pi_B^f(q) \gg \Pi_B^b(q). \quad (6)$$

We therefore have

$$\Pi_B^f(q) + \Pi_B^b(q) \approx \Pi_B^f(q), \quad (7)$$

which leads to

$$D_{\mu\nu}(q) = \frac{A_{\mu\nu}(q)}{\Pi_A^f(q) + \xi} + \frac{B_{\mu\nu}(q)}{\Pi_B^f(q)}. \quad (8)$$

In the Landau gauge, the leading order contribution of fermions to the vacuum polarization tensor is

$$\Pi_{\mu\nu}^f(q) = -N \int \frac{d^3 k}{(2\pi)^3} \text{Tr}[\gamma_{\mu} S_0(k) \gamma_{\nu} S_0(p)], \quad (9)$$

where $p = k + q$ and the free propagator of fermion is

$$S_0(k) = \frac{1}{i v_{\mu} \gamma^{\mu} k_{\mu}} = \frac{1}{i(\gamma^0 k_0 + v \gamma \cdot \mathbf{k})}. \quad (10)$$

It is straightforward to obtain

$$\Pi_A^f(q) = \Pi_B^f(q) = \frac{N\sqrt{q_0^2 + v^2|\mathbf{q}|^2}}{8v^2}. \quad (11)$$

In order to study dynamical mass generation, one can write the following DS equation for the full fermion propagator,

$$S^{-1}(p) = S_0^{-1}(p) + \int \frac{d^3k}{(2\pi)^3} \gamma^\mu S(k) \Gamma^\nu(k, p) D_{\mu\nu}(q), \quad (12)$$

where $\Gamma^\nu(k, p)$ is the full vertex function and $D_{\mu\nu}(q)$ is the full gauge field propagator with $q = p - k$. To the leading order in $1/N$ expansion, the vertex function $\Gamma^\nu(k, p)$ is replaced by the bare vertex γ^ν . The inverse full propagator of fermion can be written as

$$S^{-1}(p) = iv_\mu \gamma^\mu p_\mu \mathcal{A}(p) + m(p), \quad (13)$$

where $\mathcal{A}(p)$ is the wave-function renormalization and $m(p)$ is the fermion mass function. Strictly speaking, one can build a set of coupled equations of $\mathcal{A}(p)$ and $m(p)$. Here, for simplicity we will only consider the equation of dynamical mass $m(p)$. However, $\mathcal{A}(p)$ can not be simply taken to be unity because the fermion velocity renormalization must be calculated from $\mathcal{A}(p)$. Our strategy here is to first calculate $\mathcal{A}(p)$ and velocity renormalization perturbatively and then substitute them into the mass equation.

Taking trace on both sides of the DS equation, we arrive at an integral equation for fermion self-energy

$$m(p) = \frac{1}{4} \int \frac{d^3k}{(2\pi)^3} \frac{m(k)}{k^2 + m^2(k)} \text{Tr}[\gamma^\mu D_{\mu\nu}(p - k) \gamma^\nu]. \quad (14)$$

Using Eq.(8) and Eq.(11), we have

$$m(p_0, |\mathbf{p}|) = \frac{8}{N} \int \frac{d^3k}{(2\pi)^3} \frac{m(k_0, |\mathbf{k}|)}{k_0^2 + v^2|\mathbf{k}|^2 + m^2(k_0, |\mathbf{k}|)} \times \left[\frac{1}{\frac{\sqrt{q_0^2 + v^2|\mathbf{q}|^2}}{v^2} + \frac{8}{N}\xi} + \frac{1}{\frac{\sqrt{q_0^2 + v^2|\mathbf{q}|^2}}{v^2}} \right], \quad (15)$$

with $q_0 = p_0 - k_0$ and $\mathbf{q} = \mathbf{p} - \mathbf{k}$.

If the DS equation for $m(p)$ has only vanishing solutions, the fermions remain massless and the Lagrangian respects the chiral symmetries $\psi \rightarrow \exp(i\theta\gamma^{3,5})\psi$, with γ^3 and γ^5 two 4×4 matrices that anticommute with γ^μ ($\mu = 0, 1, 2$). If the DS equation for $m(p)$ develops a nontrivial solution, then the originally massless fermions acquire a finite dynamical mass which breaks the chiral symmetries.

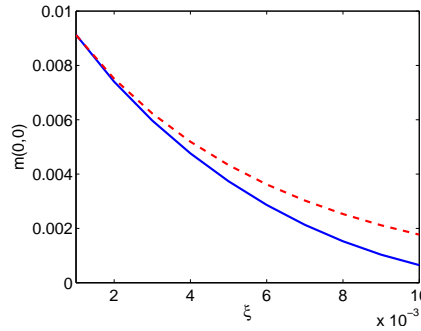


FIG. 1: The dependence of $m(0,0)$ on the parameter ξ . The cases with and without velocity renormalization are represented by solid and dashed lines, respectively.

In order to investigate the influence of fermion velocity renormalization on the dynamical mass, we now need to calculate the fermion self-energy due to gauge fluctuations. This will be done perturbatively. To the leading order of $1/N$ expansion, the self-energy is

$$\begin{aligned} \Sigma(p) &= \int \frac{d^3k}{(2\pi)^3} \gamma^\mu S_0(k) \gamma^\nu D_{\mu\nu}(p - k) \\ &= \Sigma_A(p) + \Sigma_B(p), \end{aligned} \quad (16)$$

where

$$\begin{aligned}\Sigma_A(p) &\equiv \int \frac{d^3k}{(2\pi)^3} \gamma^\mu S_0(k) \gamma^\nu \frac{A_{\mu\nu}(p-k)}{\Pi_A^f(p-k) + \xi}, \\ \Sigma_B(p) &\equiv \int \frac{d^3k}{(2\pi)^3} \gamma^\mu S_0(k) \gamma^\nu \frac{B_{\mu\nu}(p-k)}{\Pi_B^f(p-k)}.\end{aligned}\quad (17)$$

After tedious but straightforward calculations, we found that, the longitudinal contribution is

$$\begin{aligned}\Sigma_A(p) &= -\frac{4i}{N\pi^2} \gamma^0 p_0 \ln \left(\frac{\Lambda}{\max(p, \xi)} \right) \approx -\frac{4i}{N\pi^2} \gamma^0 p_0 \ln \left(\frac{\Lambda}{p + \xi} \right) \\ &= -\frac{4i}{N\pi^2} \gamma^0 p_0 \left[\ln \left(\frac{\Lambda}{p} \right) + \ln \left(\frac{p}{p + \xi} \right) \right],\end{aligned}\quad (18)$$

and the transverse contribution is

$$\Sigma_B(p) = \frac{4i}{3N\pi^2} \gamma^0 p_0 \ln \left(\frac{\Lambda}{p} \right) - \frac{8i}{3N\pi^2} v\gamma \cdot \mathbf{p} \ln \left(\frac{\Lambda}{p} \right), \quad (19)$$

with Λ the ultraviolet cutoff. Then the total self-energy can be written as

$$\Sigma(p) = \Sigma_0(p) i\gamma^0 p_0 + \Sigma_1(p) i v\gamma \cdot \mathbf{p}, \quad (20)$$

where

$$\Sigma_0(p) = -\frac{8}{3N\pi^2} \ln \left(\frac{\Lambda}{p} \right) - \frac{4}{N\pi^2} \ln \left(\frac{p}{p + \xi} \right), \quad (21)$$

$$\Sigma_1(p) = -\frac{8}{3N\pi^2} \ln \left(\frac{\Lambda}{p} \right). \quad (22)$$

It is easy to see that, the temporal component of wave function renormalization $\mathcal{A}_0(p)$ is equal to $1 + \Sigma_0(p)$ and the spatial component is $\mathcal{A}_1(p) = 1 + \Sigma_1(p)$. In the absence of non-relativistic scalar bosons, QED₃ respects the Lorentz invariance, so $\Sigma_1(p) = \Sigma_0(p)$. In the present problem, the non-relativistic scalar bosons breaks the Lorentz invariance, so that $\Sigma_1(p) \neq \Sigma_0(p)$. Indeed, the velocity renormalization is determined by their difference, $\Sigma_1(p) - \Sigma_0(p) = -\frac{4}{N\pi^2} \ln \left(\frac{p+\xi}{p} \right)$. Applying the standard renormalization group (RG) method [36, 42–44], one can approximately obtain the flow equation for fermion velocity in the low-energy regime,

$$p \frac{\partial v(p)}{\partial p} = \frac{4}{N\pi^2} v(p). \quad (23)$$

Its solution can be formally written as

$$v(p) = \left(\frac{p}{p + \xi} \right)^\eta, \quad (24)$$

where $\eta = \frac{4}{N\pi^2}$ is the anomalous dimension. It is interesting to notice that for $\xi = 0$, $v(p) = 1$, so the fermion velocity will not be renormalized when the Lorentz invariance is restored.

In order to consider the influence of fermion velocity renormalization on mass generation, we will replace the bare fermion velocity by its renormalized value. Such approach was recently employed by Khveshchenko to study an analogous issue in the context of graphene [40]. Replacing the bare fermion velocity in Eq.(15) by the renormalized velocity Eq.(24), we now have a new mass equation,

$$m(p_0, |\mathbf{p}|) = \frac{8}{N} \int \frac{d^3k}{(2\pi)^3} \frac{m(k_0, |\mathbf{k}|)}{k_0^2 + v^2(k) |\mathbf{k}|^2 + m^2(k_0, |\mathbf{k}|)} \times \left[\frac{1}{\frac{\sqrt{q_0^2 + v^2(q) |\mathbf{q}|^2}}{v^2(q)} + \frac{8}{N} \xi} + \frac{1}{\frac{\sqrt{q_0^2 + v^2(q) |\mathbf{q}|^2}}{v^2(q)}} \right], \quad (25)$$

where $q_0 = p_0 - k_0$ and $\mathbf{q} = \mathbf{p} - \mathbf{k}$.

In the numerical computation, we fix $N = 2$, corresponding to the physical flavor. In Fig.(1), the cases with and without velocity renormalization are represented by solid and dashed lines, respectively. The dynamical mass

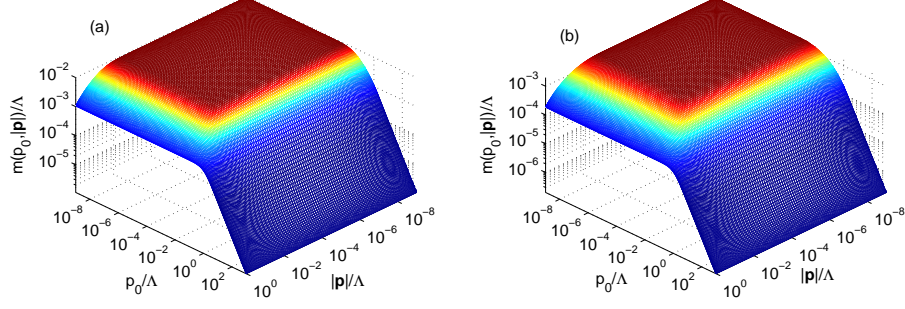


FIG. 2: Dynamical mass $m(p_0, |\mathbf{p}|)$ with (a) for $\xi/\Lambda = 10^{-3}$ and (b) for $\xi/\Lambda = 10^{-2}$.

functions $m(p_0, |\mathbf{p}|)$ in the presence of velocity renormalization are shown in Fig.(2) with (a) for $\xi/\Lambda = 10^{-3}$ and (b) for $\xi/\Lambda = 10^{-2}$. From the figures, we can draw two main conclusions. First, DCSB still happens for flavor $N = 2$ after taking into account the scalar bosons, and indeed survives when ξ is smaller than a critical value ξ_c . This indicates that it is not suitable to simply discard the temporal component of gauge field [28, 29]. Second, the fermion velocity renormalization enhances the dynamical fermion mass in the present QED₃ model. This is apparently very different from the behavior of Dirac fermion in long-range Coulomb interaction[40, 41, 43], where the interaction takes the form

$$V_C(q_0, |\mathbf{q}|) = \frac{1}{\frac{|\mathbf{q}|}{2\pi g v} + \frac{N}{8} \frac{|\mathbf{q}|^2}{\sqrt{q_0^2 + v^2} |\mathbf{q}|^2}}, \quad (26)$$

with g the coupling constant of Coulomb interaction. In the limit of infinite Coulomb repulsion $g \rightarrow \infty$, it was shown [40, 43] that the renormalized fermion velocity is

$$v(p) = \left(\frac{p}{\Lambda}\right)^{-\eta}. \quad (27)$$

Evidently, the renormalized fermion velocity is reduced in the low energy regime in our QED₃ model (see Eq.(24)), but becomes larger in the case of Coulomb interaction (see Eq.(27)). This difference leads to the different effects of velocity renormalization on dynamical mass generation in gauge-interacting and Coulomb-interacting systems.

We finally remark on the applications of our results. It is known that the high- T_c superconductor at zero doping is a Mott insulator with long-range AFM order [33]. Such long-range order persists at small doping concentration x , but is completely destroyed when $x > 0.03$ [33]. Within the effective gauge theory of high- T_c superconductor, the doping process amounts to introducing non-relativistic scalar boson ϕ , while the AFM order is represented by DCSB. In Ref. [29], it was argued that AFM order is immediately destroyed once scalar boson is present, which is not well consistent with the experimental facts. Our calculations show that DCSB can occur in the presence of scalar boson as long as parameter ξ is sufficiently small, but is destroyed when $\xi > \xi_c$. On general physical grounds, the compressibility parameter ξ should depend on the density of scalar boson (doping). Therefore, our results imply that AFM is destroyed only when doping exceeds certain critical value. This is qualitatively consistent with experimental facts.

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